Sim

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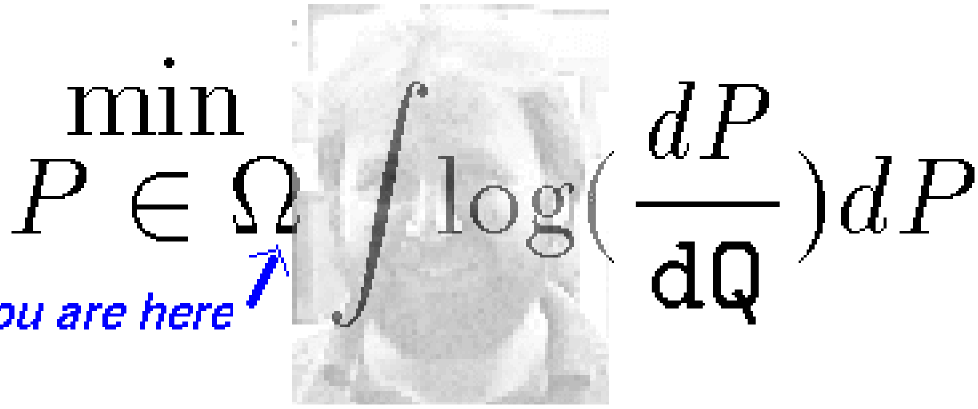
10/29/2017

Simplification by Annealing



1 Sim: Simplification by Annealing

Here I try to explain how to use “Sim”,
to automatically simplify expressions.
Sim(ex) tries to minimize the length of the LaTeX representation
of the expression “ex” by simulating annealing picking available
transformations at random from the list TL of predefined
transformations.
It is possible to direct the Simplificator to operate on a sub-expression
by setting the parameter ops to a given list or tuple.
It is also possible to specify a substitution dictionary with subs_dic.
The use of these parameters is illustrated with an example from
Monte Carlo simulation of a beta distribution by means of the rejection method.
A beta density with parameters α and β (red curve below)
has a triangular envelope (the blue lines below)
provided that $\alpha > 2$ and $\beta > 2$.
The triangular envelope is easily computed with sage (betaT.sage)
We want a simplified formula for the upper vertex of the triangle
that envelopes the beta density in terms of the parameters α and β.
The x coordinate of the upper vertex is xp. The simplification is *remarkable*
From length 564 to length 66 producing a beautiful formula.

$$xp = \frac{1}{a_1^{-a_1} a_2^{a_2} b_1^{b_1} b_2^{-b_2} + 1}$$

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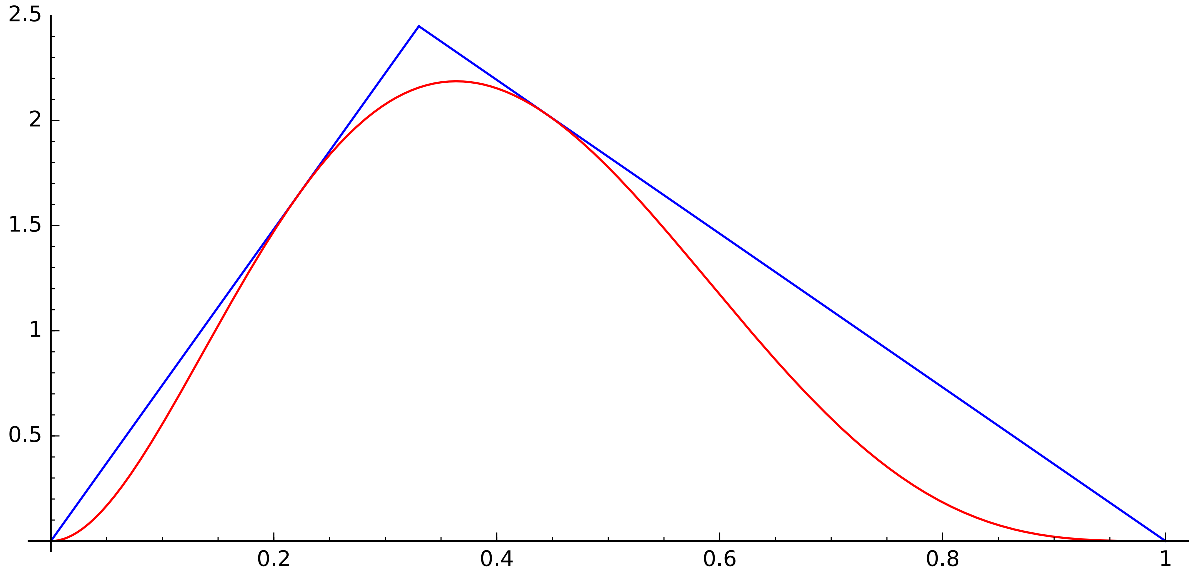
```
%typeset_mode True
%var a1,b1,a2,b2,a,c,y,z
load("http://omega.albany.edu:8008/sage/Sim.py")
load('http://omega.albany.edu:8008/sage/betaT.sage')
```

Using TL = TL_all[0:-1] = ['_rr', '_dc', '_ps', '_el', '_sl', '_fs', '_Snj', '_Sdj', '_cb', '_xp', '_sr', '_er', '_es', '_fa', '_pf']

```
p0+p1+p2
```




```
help(Sim)
```

Help on function Sim:

Sim(ex, Tmax=100, Tmin=1, steps=100, updates=0, energy=False, ops=(), subs_dic={})
    Simplification of the expression ex by Simulated Annealing.
    energy = length of the latex representation of ex.
    If either ops of subs_dic is non empty then subexpression simplification and
    substitution is applied by calling subSim.
    See also "subSim"
    Example:

    sage: var('a b c d')
    sage: e0 = sqrt((a^3-b^3)/(a-b)+a*b)
    sage: Sim(e0)

```
sage: ex = integral(1/((a*x+b)^2*(c*x+c)^2),x)
sage: ex_sim = Sim(ex,energy= True)
sage: show([ex_sim,lenl(ex)])
sage: var('a a1 b x y')
sage: t =  -2*a*b*x*y^2+3*b^2*x*y^2+(a^2*x-2*a*b*x+b^2*x)*y^2
sage: Sim(t,ops=(2,0),subs_dic={(a-b):a1},updates=5)
```

```
help(subSim)
```

```
Help on function subSim:

subSim(e, Tmax=100, Tmin=1, steps=100, updates=0, energy=False, ops=(), subs_dic={})
    subSim(e,ops=(),subs_dic={})
    subexpression simplification and substitution.
    It attempts to simplify e.op[ops] with 'Sim'.
    After simplification it applies the substitutions given by
    the subs_dic dictionary.
    It returns the original expression with the simplified subexpression and
    the substitutions.
    Example:

    sage: var('a a1 b x y')
    sage: t =  -2*a*b*x*y^2+3*b^2*x*y^2+(a^2*x-2*a*b*x+b^2*x)*y^2
    sage: subSim(t,ops=(2,0),subs_dic={(a-b):a1})
```

```
xp
```

$$\frac{\alpha \left(\frac{\alpha}{\alpha+\beta-3} - \frac{1}{\alpha+\beta-3}\right)^{\alpha-1} \left(-\frac{\alpha}{\alpha+\beta-3} + \frac{1}{\alpha+\beta-3} + 1\right)^{\beta-1}}{\beta \left(\frac{\alpha}{\alpha+\beta-3} - \frac{2}{\alpha+\beta-3}\right)^{\alpha-1} \left(-\frac{\alpha}{\alpha+\beta-3} + \frac{2}{\alpha+\beta-3} + 1\right)^{\beta-1} + \alpha \left(\frac{\alpha}{\alpha+\beta-3} - \frac{1}{\alpha+\beta-3}\right)^{\alpha-1} \left(-\frac{\alpha}{\alpha+\beta-3} + \frac{1}{\alpha+\beta-3} + 1\right)^{\beta-1}}$$

```
x1=Sim(xp)
x1
```

$$\frac{}{(\alpha-1)^{\alpha}\alpha^2(\beta-2)^{\beta}\beta + (\alpha-2)^{\alpha}\alpha(\beta-1)^{\beta}\beta^2 - (\alpha-1)^{\alpha}\alpha^2(\beta-2)^{\beta} - 4\,(\alpha-2)^{\alpha}\alpha(\beta-1)^{\beta}\beta - 4\,(\alpha-1)^{\alpha}\alpha}$$

```
dic1 = {(alpha-1):a1,(alpha-2):a2,(beta-1):b1,(beta-2):b2}
```

```
x2 = Sim(x1,subs_dic=dic1)
x2
```

$$\frac{a_1^{\alpha} a_2^2 b_1 b_2^{\beta}}{(a_2^{\alpha}\alpha - a_2^{\alpha})b_1^{\beta}\beta^2 + 4\,(a_2^{\alpha}\alpha - a_2^{\alpha})b_1^{\beta} - (a_1^{\alpha}\alpha^2 - 4\,a_1^{\alpha}\alpha + 4\,a_1^{\alpha})b_2^{\beta} - \Bigl(4\,(a_2^{\alpha}\alpha - a_2^{\alpha})b_1^{\beta} - (a_1^{\alpha}\alpha^2 - 4\,a_1^{\alpha}\alpha + 4\,a_1^{\alpha})b}$$

```
x3 = Sim(x2,ops=(0,0), subs_dic=dic1)
x3
```

$$-\frac{a_1^{\alpha} a_2^2 b_1 b_2^{\beta}}{a_1^{\alpha} a_2^2 b_2^{\beta} - (a_2^{\alpha}\alpha - a_2^{\alpha})b_1^{\beta}\beta^2 - 4\,a_1 a_2^{\alpha} b_1^{\beta} + \left(4\,(a_2^{\alpha}\alpha - a_2^{\alpha})b_1^{\beta} - (a_1^{\alpha}\alpha^2 - 4\,a_1^{\alpha}\alpha + 4\,a_1^{\alpha})b_2^{\beta}\right)\beta}$$

```
x4 = Sim(x3, ops=(0,0,1), subs_dic=dic1)
x4
```

$$\frac{a_1^{\alpha} a_2^2 b_1 b_2^{\beta}}{a_1 a_2^{\alpha} b_1^{\beta} \beta^2 - a_1^{\alpha} a_2^2 b_2^{\beta} + 4\, a_1 a_2^{\alpha} b_1^{\beta} - \left(4\,(a_2^{\alpha}\alpha - a_2^{\alpha}) b_1^{\beta} - (a_1^{\alpha}\alpha^2 - 4\, a_1^{\alpha}\alpha + 4\, a_1^{\alpha}) b_2^{\beta}\right)\beta}$$

```
x5 = Sim(x4, ops=(0,0,3,0), subs_dic=dic1)
x5
```

$$\frac{a_1^{\alpha} a_2^2 b_1 b_2^{\beta}}{a_1 a_2^{\alpha} b_1^{\beta} \beta^2 - a_1^{\alpha} a_2^2 b_2^{\beta} + 4\, a_1 a_2^{\alpha} b_1^{\beta} + \left(a_1^{\alpha} a_2^2 b_2^{\beta} - 4\, a_1 a_2^{\alpha} b_1^{\beta}\right)\beta}$$

```
x6 = Sim(x5, ops=(0,0), subs_dic=dic1)
x6
```

$$\frac{a_1^{\alpha} a_2^2 b_1 b_2^{\beta}}{a_1^{\alpha} a_2^2 b_2^{\beta} \beta + (\beta^2 + 4) a_1 a_2^{\alpha} b_1^{\beta} - a_1^{\alpha} a_2^2 b_2^{\beta} - 4\, a_1 a_2^{\alpha} b_1^{\beta} \beta}$$

```
x7 = Sim(x6, ops=(0,0), subs_dic=dic1)
x7
```

$$\frac{a_1^{\alpha} a_2^2 b_1 b_2^{\beta}}{a_1 a_2^{\alpha} b_1^{\beta} b_2^2 + a_1^{\alpha} a_2^2 b_2^{\beta} \beta - a_1^{\alpha} a_2^2 b_2^{\beta}}$$

```
x8 = Sim(x7, ops=(0,0), subs_dic=dic1)
x8
```

$$\frac{a_1^{\alpha} a_2^2 b_1 b_2^{\beta}}{a_1^{\alpha} a_2^2 b_1 b_2^{\beta} + a_1 a_2^{\alpha} b_1^{\beta} b_2^2}$$

```
ix8 = Sim(1/x8)
```

```
ix8
```

$$a_1^{-\alpha+1} a_2^{\alpha-2} b_1^{\beta-1} b_2^{-\beta+2} + 1$$

```
dic1.update({(-alpha+1):-a1,(-beta+2):-b2})
dic1
```

$$\{-\alpha + 1 : -a_1, \alpha - 1 : a_1, \beta - 1 : b_1, -\beta + 2 : -b_2, \alpha - 2 : a_2, \beta - 2 : b_2\}$$

```
x9 = Sim(ix8, subs_dic=dic1)
x9
```

$$a_1^{-a_1} a_2^{a_2} b_1^{b_1} b_2^{-b_2} + 1$$

```
# Final formula: for xp

xp_simplified = 1/x9
xp_simplified
```

$$\frac{1}{a_1^{-a_1} a_2^{a_2} b_1^{b_1} b_2^{-b_2} + 1}$$

```
lenl(xp)
lenl(xp_simplified)
```

564
66